APPENDIX B

POISSON DISTRIBUTION

B-1. Poisson Distribution. Events that occur randomly in time, and have an equally likely probability of occurrence in any unit time increment Δt (typically one year) can be described by Poisson's distribution. Events that have been assumed to follow Poisson's distribution include random arrivals (of floods, earthquakes, traffic, customers, phone calls, etc.). The Poisson distribution is a discrete distribution function that defines the probability that x events will occur in an interval of time t:

$$\text{Pr (x events in interval t)} = \Pr(x) = \frac{(\lambda t)^x e^{-\lambda t}}{x!} \qquad (1)$$

In Equation 1, λ is the mean rate of occurrence (or the expected number of events in a unit time period, Δt), the number of unit time increments is denoted by t, and the number of events is denoted by x.

$$\lambda = \frac{\textit{total number of events}}{\Delta t} \qquad (2)$$

B-2. Poisson Distribution Example. Routine maintenance is scheduled at one of the Corps of Engineers reservoir projects. One item of work is that the tainter gates in the gated spillway are to be painted. To complete this task, the upstream bulkheads need to be installed to keep the work area dry. The top of the bulkheads is Elevation 424.5. This work is to take place over a 6-month time period. Use the Poisson distribution to calculate the probability that water will not go over the bulkheads while the tainter gates are being painted. Also calculate the probability that water will go over the top of the bulkheads once or twice during painting of the tainter gates.

B-3. Solution to Problem.

Using the pool elevation frequency curve in Figure B-1, the top of the bulkheads (a pool Elevation of 424.5) corresponds to a 10-year exceedence interval for the lake level.

λ = 1/10 = 0.10 exceedence/yr

The duration of the event is one-half of a year. Note the units of t must be in term of Δt, where in this case Δt is one year.

t = 0.5 years

λt = (0.10)(0.5) = 0.05 exceedence (the expected number in the interval)

The probability of zero events (over topping of the bulkheads) occurring in the six-month period (the duration of the painting contract) is

$$\Pr[0] = \frac{(0.05)^0 e^{-0.05}}{0!} = \frac{(1)(0.9512294)}{1} = 0.951$$

Pr[0] = 95.1 percent

The probability of one event (over topping of the bulkheads one time) occurring in the six-month period (the duration of the painting contract) is

$$\Pr[1] = \frac{(0.05)^1 e^{-0.05}}{1!} = \frac{(0.05)(0.9512294)}{1} = 0.047$$

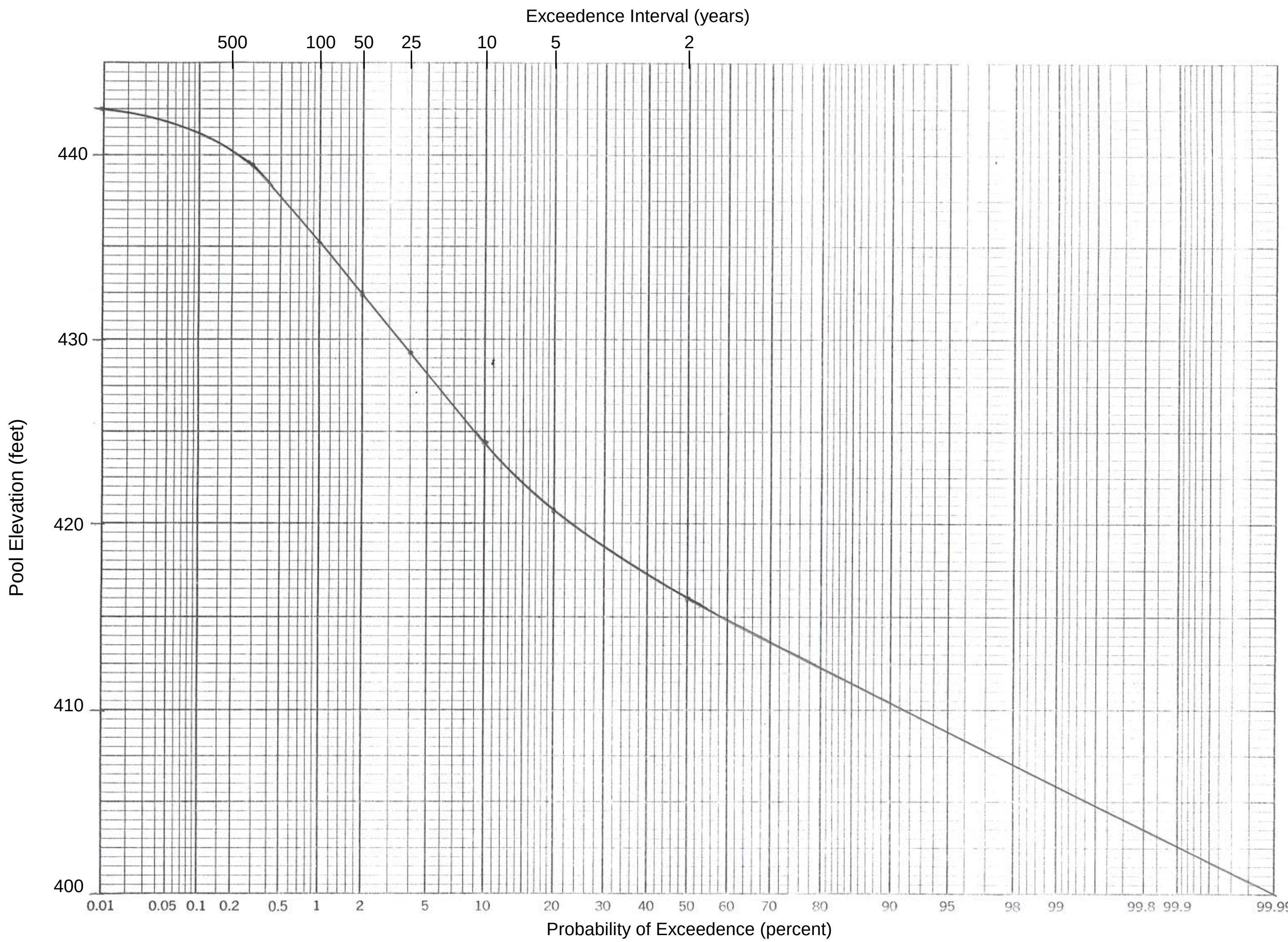
Pr[1] = 4.7 percent

The probability of two evens (over topping of the bulkheads two times) occurring in the six-month period (the duration of the painting contract) is

$$\Pr[2] = \frac{(0.05)^2 e^{-0.05}}{2!} = \frac{(0.0025)(0.9512294)}{2} = 0.001$$

Pr[2] = 0.1 percent

From the above calculation it can be seen that there is a 95% probability that the work can be accomplished without delay to the contractor by water overtopping the bulkheads. Likewise, there is a 5% probability that the lake level will rise above the top of the bulkheads and the painting of bulkheads will be delayed. There is also a 0.1% chance that the water level will overtop the bulkheads twice during the six-month painting contract.



Pool Elevation Frequency Curve

Figure B-1